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CHARACTERIZATION AND OPTIMIZATION OF ELECTROOPTIC SAMPLING BY VOLUME-INTEGRAL-METHOD AND APPLICATION OF SPACE-HARMONIC POTENTIAL

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ABSTRACT

The relationship between the electric field-components of planar microwave structures and optical fields of a Gaussian sampling-beam of an Electrooptic-Sampling-System for the case of direct probing or the use of an electrooptic probe tip are essential for the application of noncontact and non-invasive measurement of high-frequency integrated microwave circuits. The described volume-integral-method yields a rigorous treatment of the influence of the electrical field and the optical beam. In case of an external electrooptic probe tip a layered structure with a space-harmonic potential is investigated in detail and results on sensitivity and spatial resolution are presented.

INTRODUCTION

We describe a method for the calculation of the sensitivity and the spatial resolution in electrooptic sampling. The method can be applied to an external electrooptic probing tip and the direct probing in electrooptic active substrates. The determination of change in polarization and the resulting intensity variations of the reflected sampling beam after passing a polarizer is based on the rigorous application of the volume-integral-method. The Gaussian nature of the probing beam is taken into consideration in the described method.

VOLUME-INTEGRAL-METHOD

The general description of the interaction of any optical field and a microwave field can be derived by applying a perturbation ansatz to the Lorentz-Reziprocity-Theorem [1]. The results [2],[3] of this approach can be expressed as a change of optical transmission parameter. Due to polarization analysis within the measurement setup we obtain an expression for the electrooptic induced change in transmission. After some lengthy calculation we can express the entire intensity variation as follows:

$$\overline{\Delta I} = \int_0^l \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2} \boldsymbol{K}^T \boldsymbol{E}^m_{x_s} \left\{ g_1(\xi_1) + g_1(\xi_2) \right\} \; dx_s\, dy_s\, dz_s \quad (1)$$

with
$$g_1(\xi) = \frac{2}{\pi} \frac{\exp\left(-2\frac{x_s^2+y_s^2}{w^2(\xi)}\right)}{w^2(\xi)} \quad (2)$$
$$\xi_1 = 2\frac{z_s - z_0}{L} \quad ; \quad \xi_2 = 2\frac{-z_s - z_0}{L}$$

Eq. (1) describes the electrooptic induced intensity variation for a round trip of a perpendicular beam in the substrate or electrooptic probing tip. The sensitivity vector is dependant on the Pockel-coefficients of the material and orientation between optical system (Eigenmode-system) and crystal system and is denoted by $\boldsymbol{K}^T$. The microwave field is denoted $\boldsymbol{E}^m_{x_s}$, the normalized power density distribution of incident and reflected beam $g_1(\xi_1)$ and $g_1(\xi_2)$ and the confocal length is described by L. The substrate coordinate system is given by x_s, y_s, z_s.

EVALUATION OF SPACE-HARMONIC POTENTIAL

As an example we apply the volume-integral-method to a space harmonic potential for the quasi static case, characterized by the value of the phase constant= 0 :
The fields of planar microwave transmission lines can be expressed as a gradient of a scalar potential for sufficiently low frequencies and by neglecting longitudinal field components: $\boldsymbol{E}^m = \operatorname{grad}\phi$. The coordinate system is shown in Fig. 1. The electrooptic layer as well as the lateral periodic transmission line structure is assumed to be open. The layer with a relative dielectric constant ϵ_{r2} is embedded between the top and bottom layers, ϵ_{r1} and ϵ_{r3}, respectively. Width and distance of the transmission lines is given by w. With the additional assumption of an exact cosine shape of the field components in x-direction the Helmholtz-equation of the Hertzian vector reduces to the Laplace-equation for the scalar potential. For further calculation we apply the following potential to the three layers:

$$\phi(x,z) = \phi_i \cos\alpha_0 x \ldots \mathrm{i}=1,2,3 \quad (3)$$
$$\phi_1 = A\,\mathrm{e}^{-\alpha_0(z+d)}$$

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$$\phi_2 = B\,e^{-\alpha_0 z} + C\,e^{\alpha_0 z}$$
$$\phi_3 = G\,e^{-\alpha_0 z} - H\,e^{\alpha_0 z}$$
$$\alpha_0 = \frac{\pi}{2}\frac{1}{w}$$

For a forced potential:

$$\phi(x, z=-h) = V_0 \cos \alpha_0 x \tag{4}$$

at $z=-h$ and after some calculation we can determine the field components E_{x2} and E_{z2} in the EO-active layer 2:

$$E_{x2} = -\alpha_0 C(\alpha_0)\left(-K\,e^{-\alpha_0 z}\,e^{-2\alpha_0 h} + e^{\alpha_0 z}\right)\sin \alpha_0 x \tag{5}$$
$$E_{z2} = \alpha_0 C(\alpha_0)\left(K\,e^{-\alpha_0 z}\,e^{2\alpha_0 h} + e^{\alpha_0 z}\right)\cos \alpha_0 x \tag{6}$$

$$\text{with} \quad C(\alpha_0) = \frac{V_0\,e^{-\alpha_0 d}}{1-K\,e^{2\alpha_0 h}}\left(1+\frac{2}{e^{-\alpha_0 d} - \frac{S_1}{S_2}}\right) \tag{7}$$
$$K = \frac{\epsilon_{r1}+\epsilon_{r2}}{\epsilon_{r1}-\epsilon_{r2}}$$
$$S_1 = 1 + \frac{\epsilon_{r2}}{\epsilon_{r3}}\cdot\frac{1+K\,e^{2\alpha_0 h}}{1-K\,e^{2\alpha_0 h}}$$
$$S_2 = -1 + \frac{\epsilon_{r2}}{\epsilon_{r3}}\cdot\frac{1+K\,e^{2\alpha_0 h}}{1-K\,e^{2\alpha_0 h}}$$

Due to the assumption $E_y^m \doteq 0$ the integration in Eq. (1) can be performed with respect to y_s. As an example for evaluation of the analysis we consider the purely longitudinal case of probing which is possible for GaAs, and for (3m)-crystals like $LiTaO_3$ and $LiNbO_3$ with a 35° crystal cut: $K_x = K_y = 0$, and $K_z \neq 0$.

We obtain the intensity variation of ΔI at the probing position X to:

$$\overline{\Delta I}(X) = \int_0^d \int_{-\infty}^{\infty} K_z E_z^m(x,z)\ \{\ g_2(x-X,\xi_1) + g_2(x-X,\xi_2)\ \}\ dx\,dz \tag{8}$$

$$\text{with} \quad g_2(x,\xi_\imath) = \sqrt{\frac{2}{\pi}}\cdot\frac{\exp\left(-2\frac{x^2}{w^2(\xi_\imath)}\right)}{w(\xi_\imath)} \tag{9}$$

Applying fourier transformation to obtain the spectral domain representation the inner product of Eq. (9) can be expressed as the product of:

$$\widetilde{E_z^m}(\alpha) = FT\{E_z^m(x_s)\} \text{ and} \tag{10}$$
$$\widetilde{g_2}(\alpha,\xi_j) = FT\{g_2(x_s,\xi_j)\}\ldots j=1,2 \tag{11}$$

and we obtain:

$$\overline{\Delta I}(\alpha) = K_z \int_0^d \widetilde{E_z^m}\{\widetilde{g_2}(\alpha,\xi_1) + \widetilde{g_2}(\alpha,\xi_2)\}\ dz \tag{12}$$

In the following we define a transmission characteristic (normalized with respect to $\phi(x, z=-h)$):

$$H(\alpha) = \frac{\overline{\Delta I}(\alpha)}{\tilde{\phi}(\alpha, z=-h)} \tag{13}$$

After Fourier transform of the field component E_z^m and g_2 as well as the potential $\phi(x, z=-h)$ we obtain

$$H(\alpha) = K_z\cdot\alpha\cdot C(\alpha)\cdot \int_0^d \left(k\,e^{-\alpha z}\,e^{2\alpha h} - e^{\alpha z}\right)\left(e^{-\frac{\alpha^2 w^2(\xi_1)}{8}} + e^{-\frac{\alpha^2 w^2(\xi_2)}{8}}\right) dz \tag{14}$$

Since α_0 equals the local phase constant we set the decaying factor $\alpha = \alpha_0$ in Eq. (15). The results of $H(\alpha)/K_z$ are shown in the following diagrams for different parameters. As an example we choose GaAs with $n = 3.41$, $\lambda = 1.3\,\mu$m and if not mentioned differently, $\epsilon_{r_1} = 1$, $\epsilon_{r_2} = 13$, $\epsilon_{r_3} = 1$).

SENSITIVITY OF EXTERNAL ELECTROOPTIC PROBING TIP

The transmission function H(α) exhibits a bandpass characteristic. The maximum $H_{\max}$ is between the space-frequency h^{-1} und d^{-1}. Thus a reduction of sensitivity is caused by h and d. In practice h must be chosen smaller than d. The sensitivity of the probe is determined only by the optical characteristics (w_0, z_0) if the absolute value of h is small. The lower 3 dB frequency is determined by d and therfore should be chosen as large as possible.

The transmission function depends on geometrical and optical parameters which are described as follows:

- **Probe distance h:**
  The sensitivity |H(α)| as well as the spatial resolution are significantly dependant on the probe distance h (Fig. 2). The maximum of |H(α)| at $h = 1\,\mu$m compared to $h = 0.1\,\mu$m reduces by a factor of 10, from approximately 250 (curve 1) to 25 (curve 2). The sensitivity is reduced by a factor of 2 ($w_0 = 4\,\mu$m, $h = 10\,\mu$m, $z_0 = 0$, $d = 30\,\mu$m) for a spatial frequency of $\alpha = 1.12\cdot10^5$ m^{-1} which corresponds to a line spacing of 14 μm. The slope of $H(\alpha)$ at higher spatial frequencies is 10 dB/dec. for $h = 0.1\,\mu$m and 22 dB/dec. at $h = 5\,\mu$m.
- **Optical parameters w_0, z_0:**

An increase of minimal Gaussian radius w_0 has a significant effect at higher spatial-frequencies: An increase of $w_0 = 2\,\mu$m to $w_0 = 5\,\mu$m at $\alpha = 9 \cdot 10^5\,\mathrm{m}^{-1}$ results in a reduction of sensitivity by a factor of 10. The position z_0 of the waist of the Gaussian beam within the EO-crystal only influences the spatial resolution for typical EO-substrate thickness of 30...100 μm (Fig. 3 and 5) at higher spatial-frequencies.

- **Probe thickness d:**
  An increase in electrooptic probe thickness d from $30\,\mu$m to $100\,\mu$m results in an increase of sensitivity of only 1.6 (Fig. 4). The thickness d has no influence at higher spatial frequencies.

- **Spatial resolution:**
  The maximum spatial resolution (α at 3dB decrease of $H(\alpha)$) is for ideal conditions ($w_0 = 2\,\mu$m, $z_0 = 0$, $h = 1\,\mu$m, $d = 30\,\mu$m) $1.5 \cdot 10^5\,\mathrm{m}^{-1}$ which corresponds to a line width of $w = 10.5\,\mu$m. For identical parameters but a probe distance of $h = 0.1\,\mu$m we obtain a minimal line width of $w = 4.9\mu$m.

- **Influence of the dielectric constant of layer 1:**
  As shown in Fig. 6 the dielectric constant of layer 1 only influences the sensitivity at very small spatial frequencies. This is caused by the significant decay of the field components in the EO-crystal at higher spatial frequencies. The transmission characteristic $|H(\alpha)|$ for $LiTaO_3$ ($\epsilon_{r_2} = 42$, curve 2,3) exhibits a higher maximum value compared to GaAs ($\epsilon_{r_2} = 13$, curve 1,4) due to the higher dielectric constant.

CONCLUSION

The presented volume-integral-method can be applied to the case of external and internal electrooptic probing and yields important results on the introduced transmission characteristic which is described by sensitivity and spatial resolution. For the case of external electrooptic sampling the influence of the probing beam parameters and geometric dimensions of probing tip and its distance to the planar circuit is presented. The described method can be applied for effective optimization of the electrooptic sampling setup for various probing geometries.

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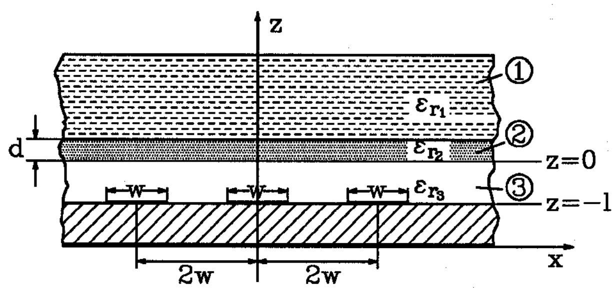

Figure 1: Transversal periodic transmission lines with electrooptic layer for determination of the space harmonic potential.

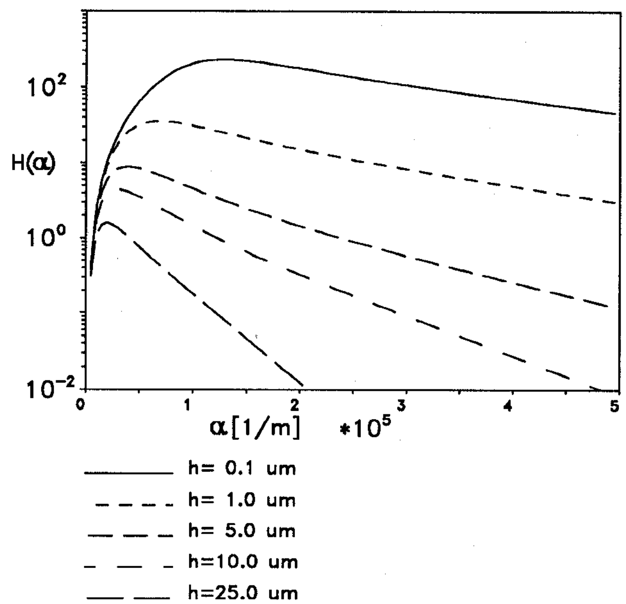

Figure 2: Transmission characteristic $H(\alpha)$ with probe distance h as parameter, and with $w_0 = 2\,\mu$m, $z_0 = 0$, electrooptic crystal thickness $d = 30\,\mu$m.

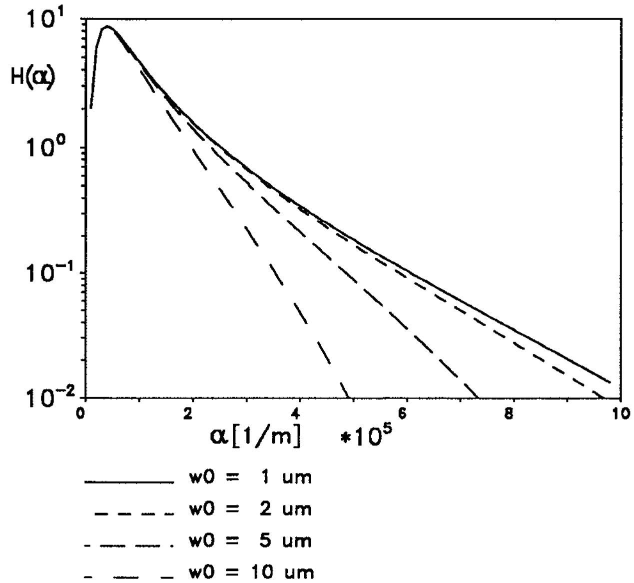

Figure 3: Transmission characteristic $H(\alpha)$ with minimal Gaussian beam radius w_0 as parameter, $z_0 = 0$, EO-thickness $d = 30\,\mu$m, $h = 5\,\mu$m.

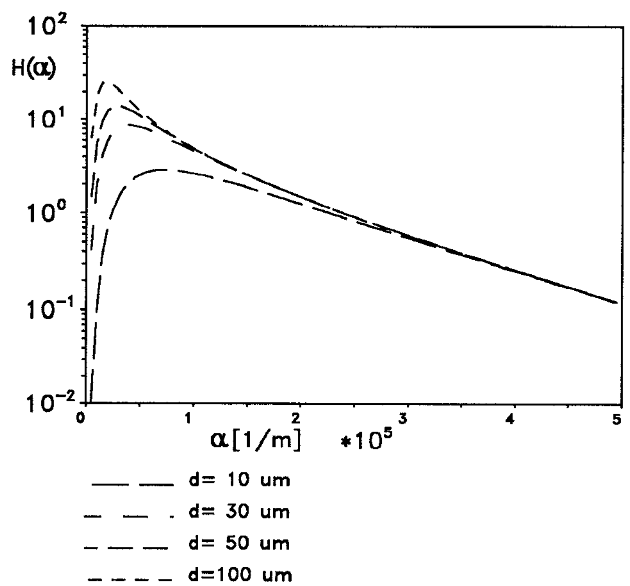

Figure 4: Transmission characteristic $H(\alpha)$ with probe thickness d as parameter, and with $w_0 = 2\,\mu$m, $z_0 = 0$, $h = 5\,\mu$m.

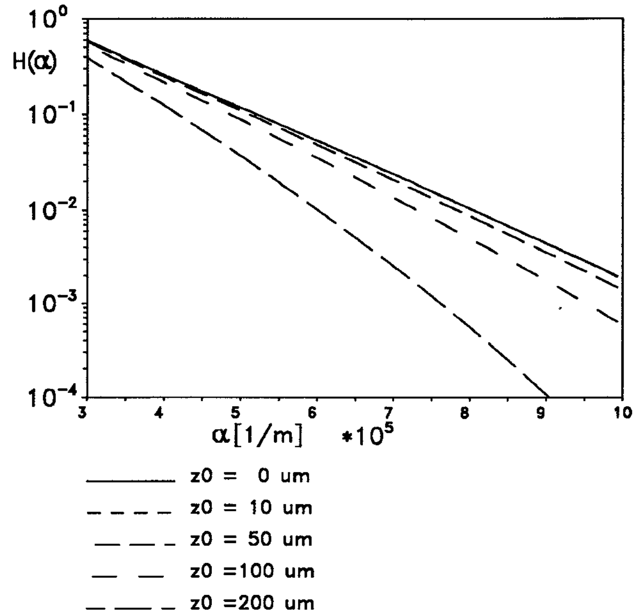

Figure 5: Transmission characteristic $H(\alpha)$ with z_0 as parameter, $w_0 = 2\,\mu$m, $d = 200\,\mu$m, $h = 5\,\mu$m.

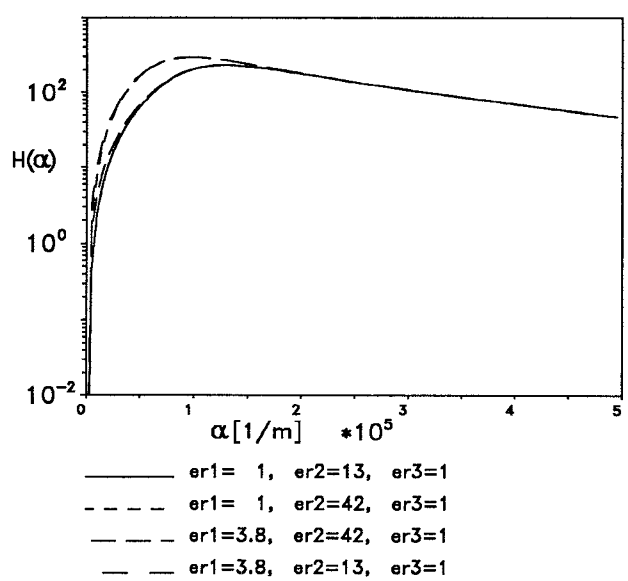

Figure 6: Transmission characteristic $H(\alpha)$ with different dielectric constants $\epsilon_1, \epsilon_2, \epsilon_3$ of the layers, and with $d = 30\,\mu$m, $w_0 = 2\,\mu$m, $z_0 = 0$, $h = 0.1\,\mu$m.